To:
The Commission of Academic Degrees
Universíty of Łódź

November 20, 2021

Review of the scientific achievments of Justyna Szpond for the habilitation committee

General Remarks

The scientific achievments by Justyna Szpond described in the document *Professional Accomplishmets* consists seven research papers

1. *Unexpected hypersurfaces with multiple fat points* (DOI:10.1016/j.jsc.2020.07.018)
2. *Quartic unexpected curves and surfaces* [BMSS20]
3. *Unexpected curves and Togliatti-type surfaces*[Szp20]
4. *Fermat-type arrangements* (DOI:10.1007/978-3-030-52111-0_12)
5. *Fermat-type configurations of lines in* $\mathbb{P}^3$ *and the containment problem* [MS18a]
6. *On codimension two flats in Fermat-type arrangements*[MS18b]
7. *From Pappus Theorem to paramter spaces of some extremal line point configurations and applications*[LBS17]

Five of them have been published in well-respected international mathematical journals and two as peer-reviewed research papers in conference proceedings (5 and 6). Apart from these seven publications, Justyna Szpond has an extensive publication list spanning over several related areas of algebraic geometry. (cf Appendix) In this review I will only consider the seven publications listed above.

The containment problem

The containment problem deals with the question what symbolic powers $I^{(m)}$ that are contained in the ordinary power I^n. A surprising result by Dumnicki, Szemberg and Tutaj-Gasińska from 2013 showed that $I^{(3)}$ does not have to be contained in I^2 for ideals of poinrsi in the projective plane.

Justyna Szpond has in a series of papers following this result been investigating similar situations for configurations of points, lines and other linear subspaces in projective spaces.

In *Fermat-type configurations of lines in $\mathbb{P}^3$ and the containment problem*, the authors contributes with a new large class of examples of where the containment $I^{(3)} \subseteq I^2$ fails. The novelty is that these are given by configurations of lines in $\mathbb{P}^3$ rather than as configurations of points. These configurations are called *restricted Fermat configurations* and are interesting in themselves, not only in terms of the failure of the containment.

In *On codimension two flats in Fermat-type arrangements*, the authors prove a strong result providing a large class of examples where $I^{(3)}$ is not contained in I^2 generalizing the previous examples of points in $\mathbb{P}^2$ and lines in $\mathbb{P}^3$.

In *From Pappus Theorem to parameter spaces of some extremal line point configurations and applications*, the authors shows that the line point configurations introduced by Böröczky are not rigid and they provide a one-dimensional parameter space of such configurations. In the case of $\mathbb{B}12$, this parameter space is rational while in the case of $\mathbb{B}15$ it is not.

The existence of unexpected hypersurfaces

The study of unexpected hypersurfaces comes from the question about whether general points with given multiplicities give independent conditions on given linear systems of divisors and the terminology was introduced in 2018 by Cook II, Harbourne, Migliore and Nagel, but the phenomenon has been studied in various shapes long before that.

Justyna Szpond has made several important contributions in this field in the articles presented here.

In *Unexpected curves with multiple fat points* there is for example a results giving the explicit formula for the equation of the unexpected curve with respect to the intersection points of a Fermat arrangement in $\mathbb{P}^2$ and a general point of multiplicity 4.

In *Fermat-Type Arrangements*, which is primarily a survey, there are also some new interesting results on unexpected curves related to duals of Fermat line arrangements in the plane.

In *Quartic unexpected curves and surfaces* the authors extend the known results for the configuratoin of nine points in $\mathbb{P}^2$ dual to the Coxeter arrangement $B3$ and they also provide a detailed analysis of the unexpected quartic surface in $\mathbb{P}^3$ passing through the complete intersection given by $x_0^3 = x_1^3 = x_2^3 = x_3^3$ with a triple point at a general point.

In *Unexpected curves and Togliatti-type surfaces*, Szpond, relates the study of unexpected hypersurfaces to *Lefschetz properties* and *defective osculating spaces*. In particular, Szpond provides a deatailed analysis of the unexpected quartics that comes from the B_3 configuration of nine points in $\mathbb{P}^2$.

Conclusions

The scientific contributions by Justyna Szpond reflected in the publications described in the achievement are on an internationally high level and of an extent that constitute a significant contribution to the development of the research area in which she has been active.

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Appendix – Publication indexed by MathSciNet

The following list of publications obtained from MathSciNet.

The publications [Szp20], [BMSS20], [MS18a], [MS18b],[LBS17] are the items 3, 2, 5, 6 and 7 mentioned in the document *Professional Accomplishments*.

Publications by Justyna Szpond listed on MathSciNet

[BDRS+19] Thomas Bauer, Sandra Di Rocco, David Schmitz, Tomasz Szemberg, and Justyna Szpond. On the postulation of lines and a fat line. *J. Symbolic Comput.*, 91:3–16, 2019.

[BMMS18] Harold Blum, Grzegorz Malara, Georg Merz, and Justyna Szpond. Notes on local positivity and Newton-Okounkov bodies. In *Extended abstracts February 2016—positivity and valuations*, volume 9 of *Trends Math. Res. Perspect. CRM Barc.*, pages 105–111. Birkhäuser/Springer, Cham, 2018.

[BMSS20] Thomas Bauer, Grzegorz Malara, Tomasz Szemberg, and Justyna Szpond. Quartic unexpected curves and surfaces. *Manuscripta Math.*, 161(3-4):283–292, 2020.

[CDF+16] A. Czapliński, M. Dumnicki, Ł . Farnik, J. Gwoździewicz, M. Lampa-Baczyńska, G. Malara, T. Szemberg, J. Szpond, and H. Tutaj-Gasińska. On the Sylvester-Gallai theorem for conics. *Rend. Semin. Mat. Univ. Padova*, 136:191–203, 2016.

[CFG+20] Susan Cooper, Giuliana Fatabbi, Elena Guardo, Anna Lorenzini, Juan Migliore, Uwe Nagel, Alexandra Seceleanu, Justyna Szpond, and Adam Van Tuyl. Symbolic powers of codimension two Cohen-Macaulay ideals. *Comm. Algebra*, 48(11):4663–4680, 2020.

[DFH+20] Marcin Dumnicki, Ł ucja Farnik, Brian Harbourne, Grzegorz Malara, Justyna Szpond, and Halszka Tutaj-Gasińska. A matrixwise approach to unexpected hypersurfaces. *Linear Algebra Appl.*, 592:113–133, 2020.

[DGS20] Marcin Dumnicki, Janusz Gwoździewicz, and Justyna Szpond. An elementary, geometric proof of the nonexistence of a projective plane of order 6. *Contrib. Discrete Math.*, 15(1):1–9, 2020.

[DHS18] M. Dumnicki, D. Harrer, and J. Szpond. On absolute linear Harbourne constants. *Finite Fields Appl.*, 51:371–387, 2018.

[DZFSTG19] Marcin Dumnicki, Mohammad Zaman Fashami, Justyna Szpond, and Halszka Tutaj-Gasińska. Lower bounds for Waldschmidt constants of generic lines in $\mathbb{P}^3$ and a Chudnovsky-type theorem. *Mediterr. J. Math.*, 16(2):Paper No. 53, 15, 2019.

[FGH+17] Ł ucja Farnik, J. Gwoździewicz, B. Hejmej, M. Lampa-Baczyńska, G. Malara, and J. Szpond. Initial sequences and Waldschmidt constants of planar point configurations. *Internat. J. Algebra Comput.*, 27(6):717–729, 2017.

[FSSTG17] Ł ucja Farnik, Tomasz Szemberg, Justyna Szpond, and Halszka Tutaj-Gasińska. Restrictions on Seshadri constants on surfaces. *Taiwanese J. Math.*, 21(1):27–41, 2017.

[LBS17] Magdalena Lampa-Baczyńska and Justyna Szpond. From Pappus Theorem to parameter spaces of some extremal line point configurations and applications. *Geom. Dedicata*, 188:103–121, 2017.

[MS18a] Grzegorz Malara and Justyna Szpond. Fermat-type configurations of lines in $\mathbb{P}^3$ and the containment problem. *J. Pure Appl. Algebra*, 222(8):2323–2329, 2018.

[MS18b] Grzegorz Malara and Justyna Szpond. On codimension two flats in Fermat-type arrangements. In *Multigraded algebra and applications*, volume 238 of *Springer Proc. Math. Stat.*, pages 95–109. Springer, Cham, 2018.

[MSS18] Grzegorz Malara, Tomasz Szemberg, and Justyna Szpond. On a conjecture of Demailly and new bounds on Waldschmidt constants in $\mathbb{P}^N$. *J. Number Theory*, 189:211–219, 2018.

[SM17] Justyna Szpond and Grzegorz Malara. The containment problem and a rational simplicial arrangement. *Electron. Res. Announc. Math. Sci.*, 24:123–128, 2017.

[SS17] T. Szemberg and J. Szpond. On the containment problem. *Rend. Circ. Mat. Palermo (2)*, 66(2):233–245, 2017.

[SS18] Tomasz Szemberg and Justyna Szpond. Waldschmidt constants for Stanley-Reisner ideals of a class of graphs. In *Multigraded algebra and applications*, volume 238 of *Springer Proc. Math. Stat.*, pages 159–167. Springer, Cham, 2018.

[Szp18] Justyna Szpond. On Hirzebruch type inequalities and applications. In *Extended abstracts February 2016—positivity and valuations*, volume 9 of *Trends Math. Res. Perspect. CRM Barc.*, pages 89–94. Birkhäuser/Springer, Cham, 2018.

[Szp20] Justyna Szpond. Unexpected curves and Togliatti-type surfaces. *Math. Nachr.*, 293(1):158–168, 2020.